Six Standard Playing Cards Are Sufficient for All Three-Input Boolean Functions

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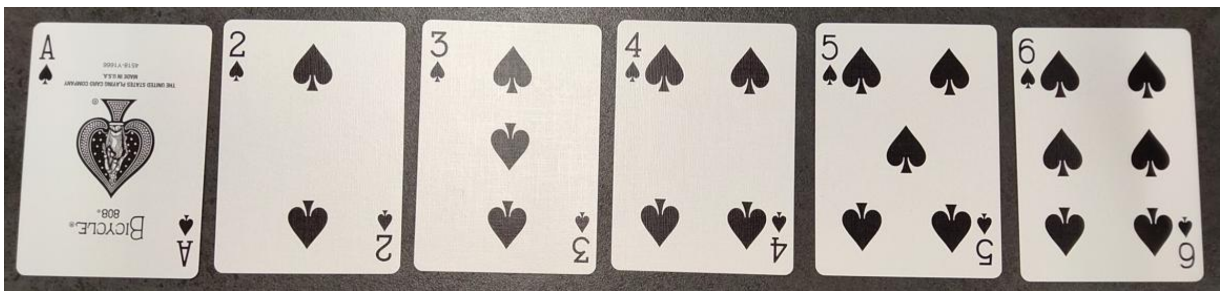
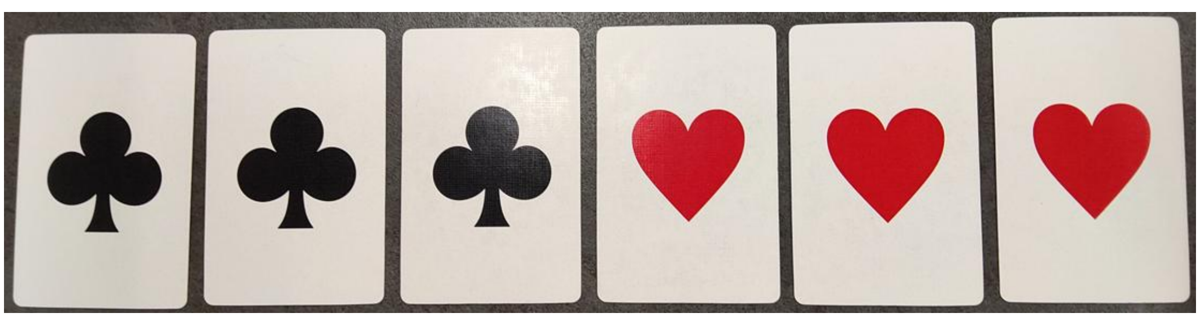
Tohoku University

Six *Standard Playing Cards* Are Sufficient for All Three-Input Boolean Functions

Standard playing cards



Two-color deck of cards



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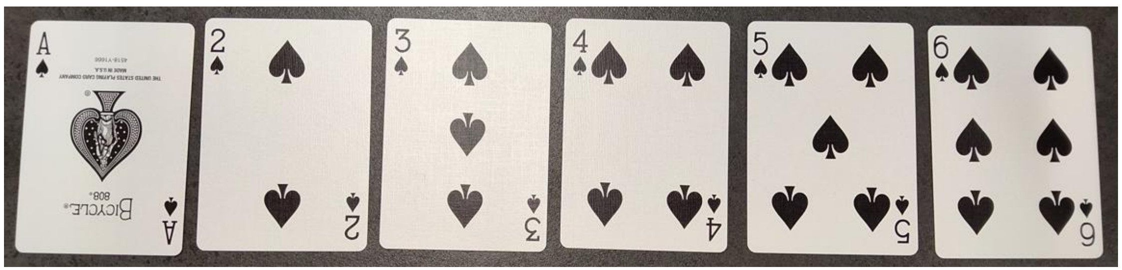
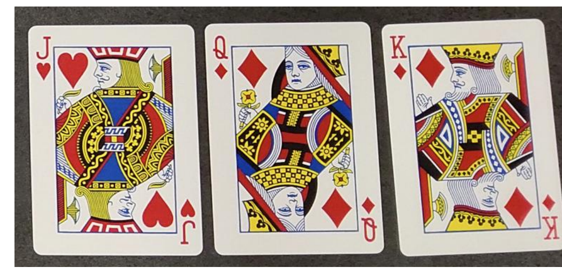
Standard playing cards



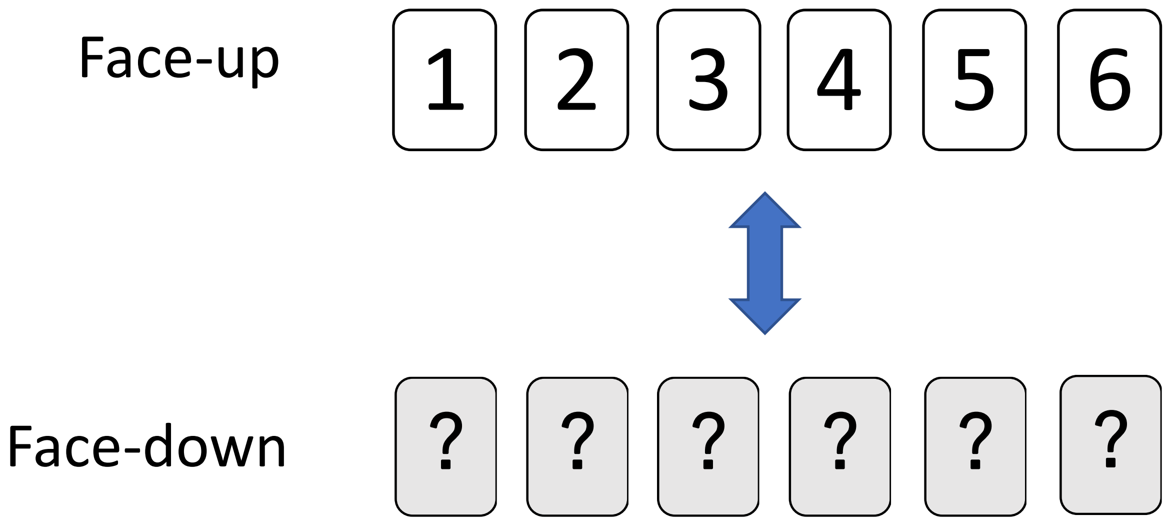
…



1 2 3 4 5 6 … 50 51 52

Numbered cards

Six Standard Playing *Cards* Are Sufficient for All Three-Input Boolean Functions



Six Standard Playing Cards Are Sufficient for All *Three-Input Boolean Functions*

$a \in \{0,1\}$

Alice

$b \in \{0,1\}$

Bob

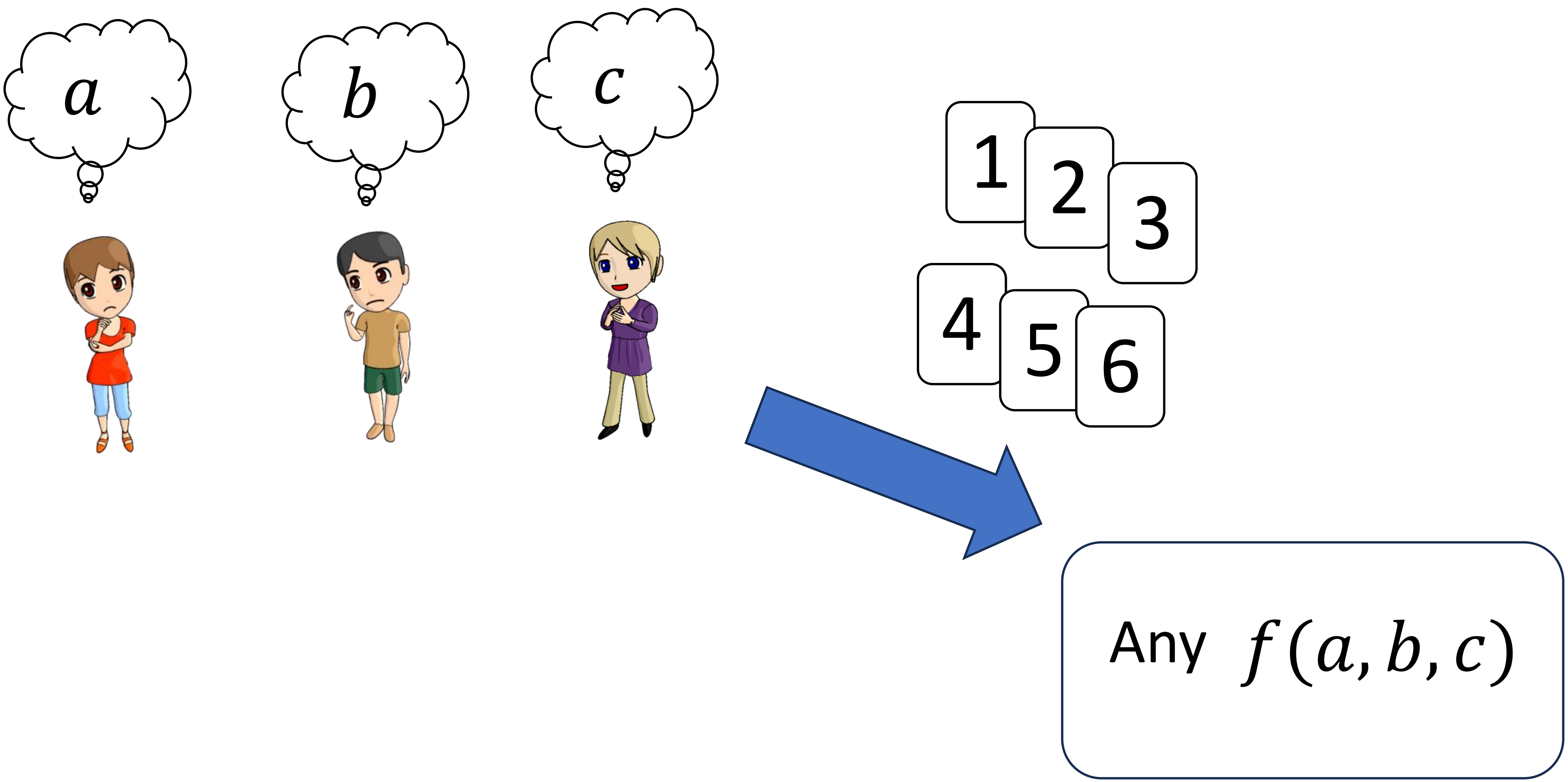
$c \in \{0,1\}$

Carol

Secure computation

$f(a, b, c)$

Six Standard Playing Cards Are Sufficient for All Three-Input Boolean Functions



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3. Analysis by NPN Equivalence classes
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Encoding and Commitment
Know Results
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Encoding

For $i < j$,

$$\boxed{i}\,\boxed{j} = 0$$

$$\boxed{j}\,\boxed{i} = 1$$

Example:

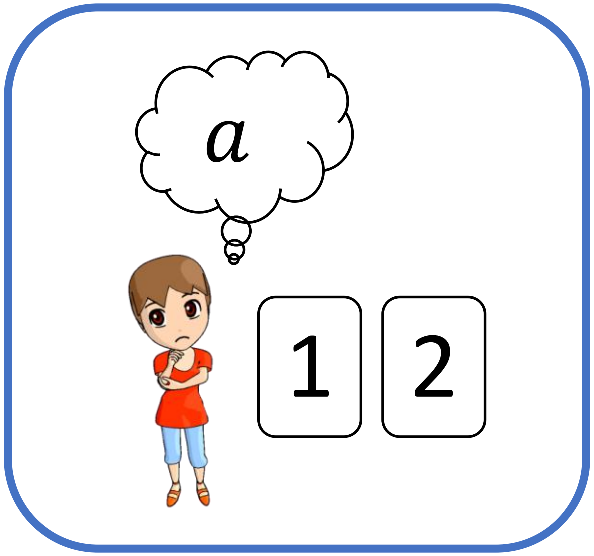
$$\boxed{1}\,\boxed{2} = 0$$

$$\boxed{2}\,\boxed{1} = 1$$

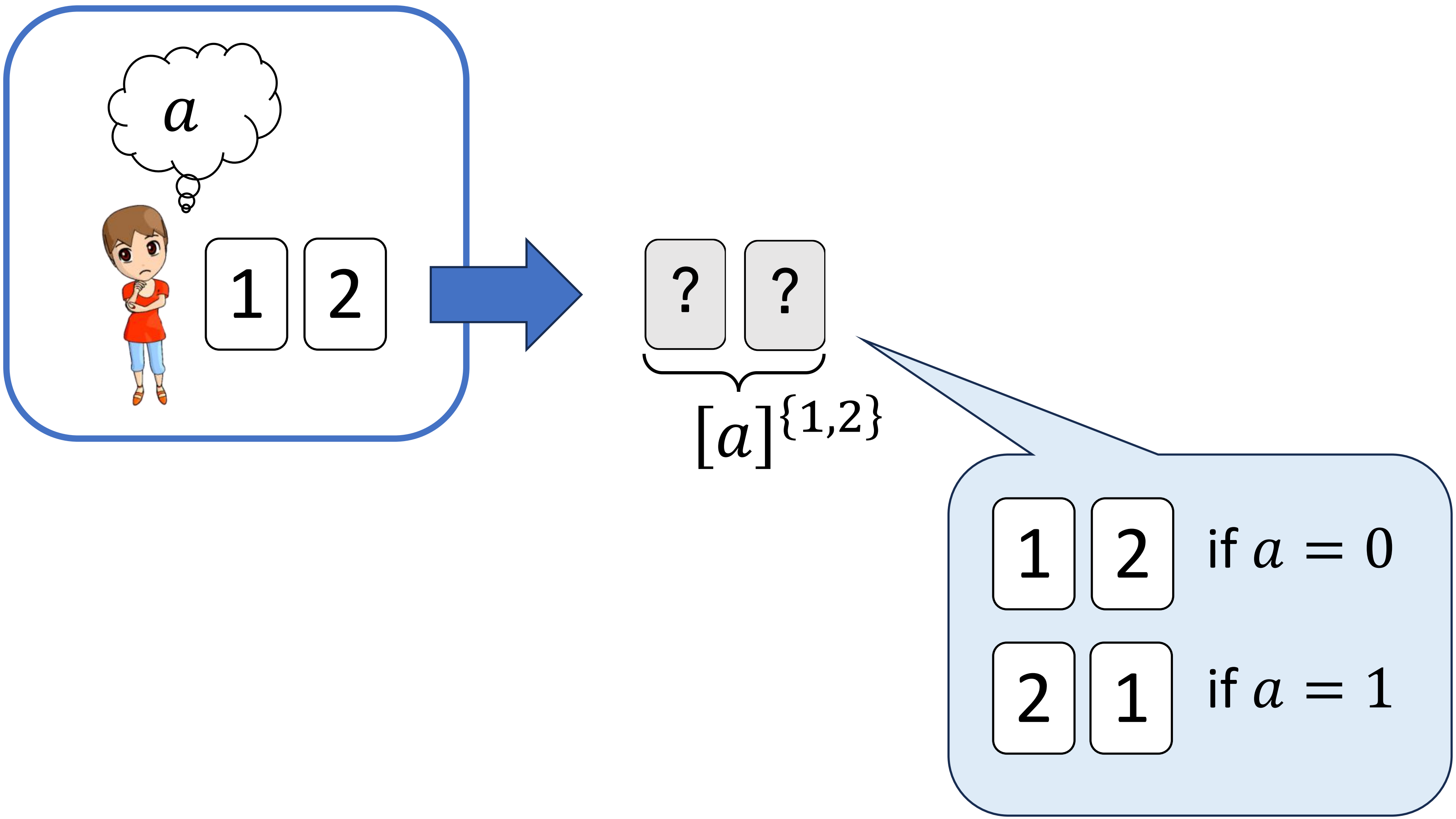
$$\boxed{1}\,\boxed{4} = 0$$

$$\boxed{4}\,\boxed{1} = 1$$

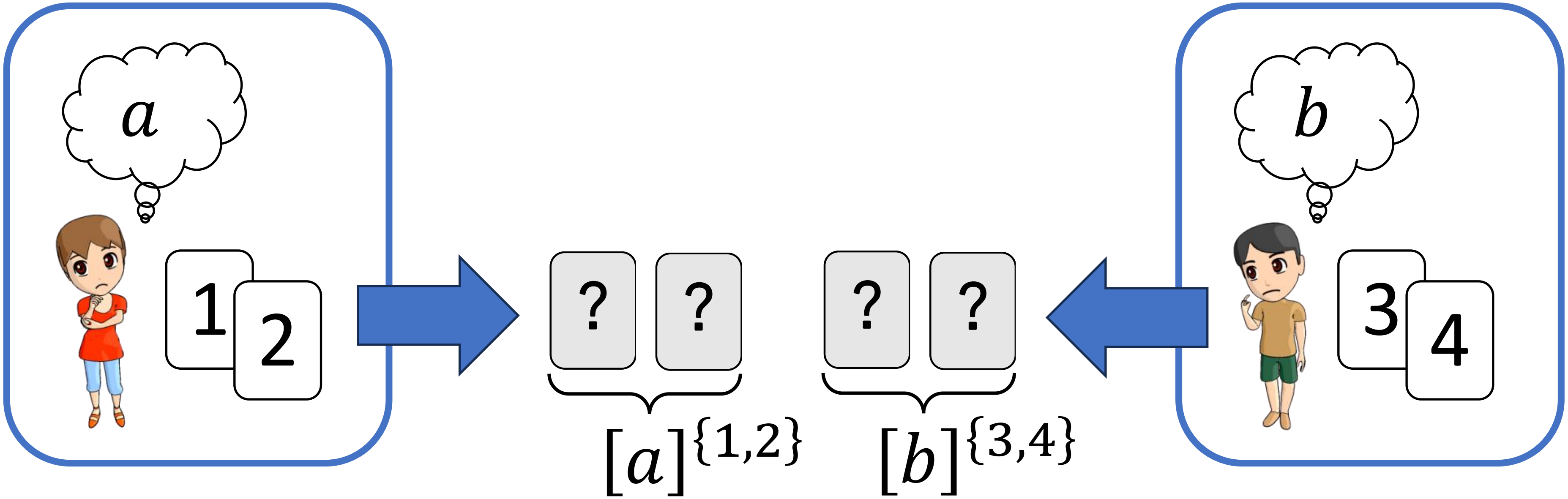
Commitment



Commitment

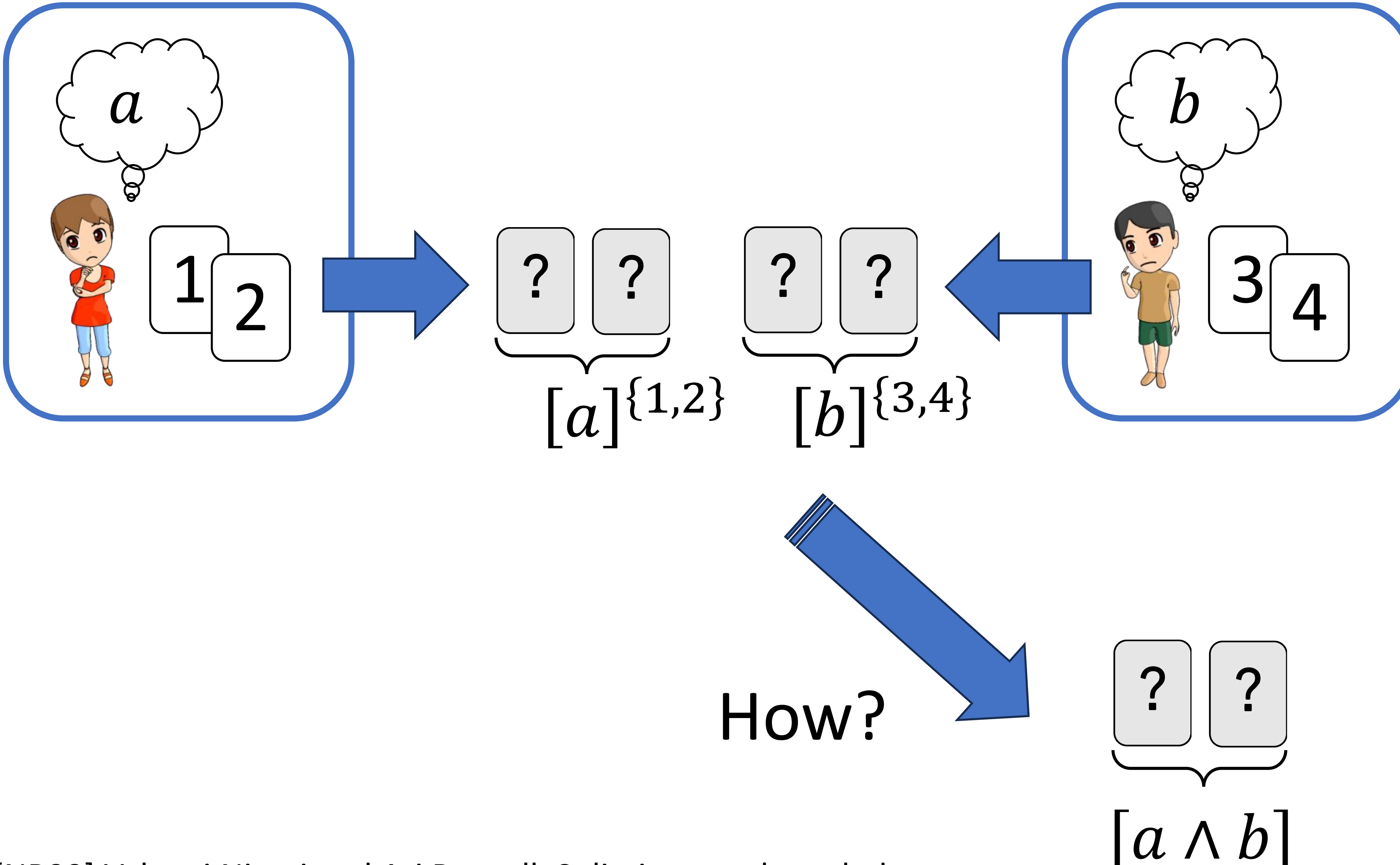


AND protocol [NR99]



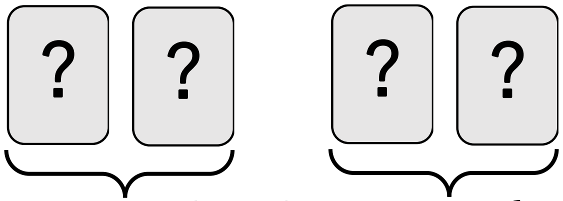
[NR99] Valtteri Niemi and Ari Renvall. Solitaire zero-knowledge. Fundam.Inf., 38(1,2):181–188, 1999

AND protocol [NR99]



[NR99] Valtteri Niemi and Ari Renvall. Solitaire zero-knowledge. Fundam.Inf., 38(1,2):181–188, 1999

Idea



$[a]^{\{1,2\}}$ $[b]^{\{3,4\}}$

$(\boldsymbol{a}, \boldsymbol{b})$				
(0,0)	1	2	3	4
(0,1)	1	2	4	3
(1,0)	2	1	3	4
(1,1)	2	1	4	3

$\underbrace{\boxed{?}\ \boxed{?}}_{[a]^{\{1,2\}}}$ $\underbrace{\boxed{?}\ \boxed{?}}_{[b]^{\{3,4\}}}$

(a,b)				
(0,0)	1	2	3	4
(0,1)	1	2	4	3
(1,0)	2	1	3	4
(1,1)	2	1	4	3

If we ignore $\boxed{2}\ \boxed{3}$

(a,b)				
(0,0)	1	2	3	4
(0,1)	1	2	4	3
(1,0)	2	1	3	4
(1,1)	2	1	4	3

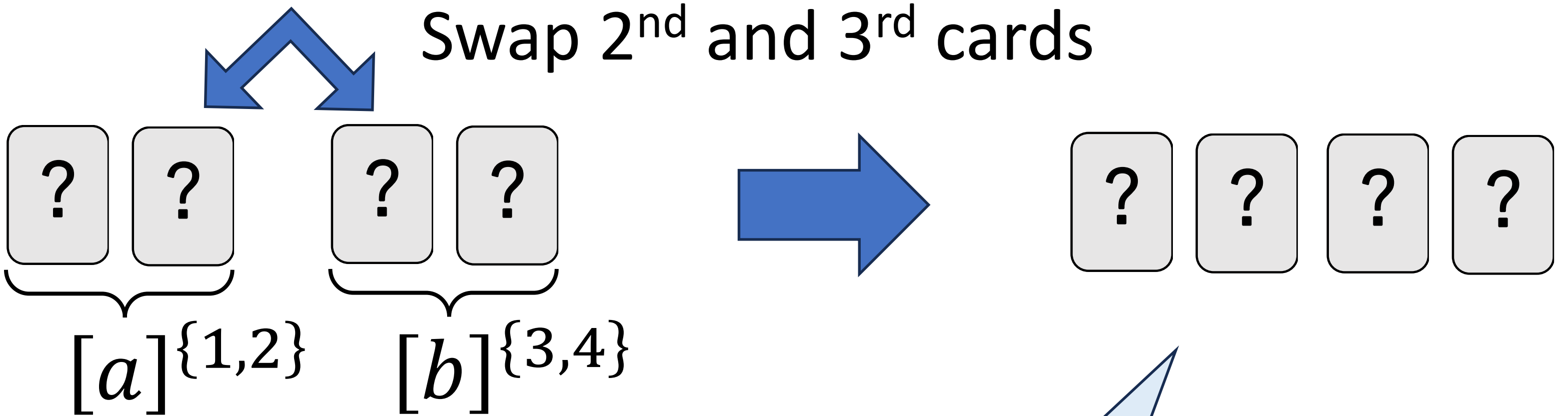
$\boxed{1}\ \boxed{4} = 0$

$\boxed{1}\ \boxed{4} = 0$

$\boxed{1}\ \boxed{4} = 0$

$\boxed{1}\ \boxed{4} = 0$

Swap 2nd and 3rd cards



(a,b)				
(0,0)	1	2	3	4
(0,1)	1	2	4	3
(1,0)	2	1	3	4
(1,1)	2	1	4	3

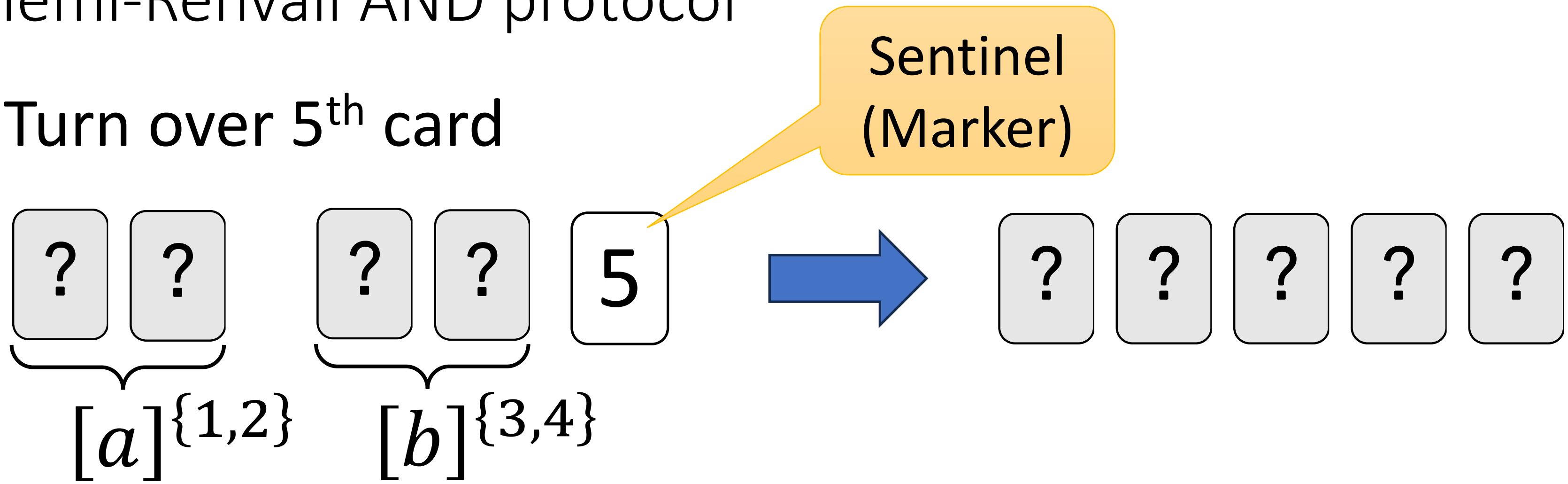
(a,b)				
(0,0)	1	3	2	4
(0,1)	1	4	2	3
(1,0)	2	3	1	4
(1,1)	2	4	1	3

$$\left.\begin{array}{l} \boxed{1}\,\boxed{4} = 0 \\ \boxed{1}\,\boxed{4} = 0 \\ \boxed{1}\,\boxed{4} = 0 \\ \boxed{4}\,\boxed{1} = 1 \end{array}\right\} a \wedge b$$

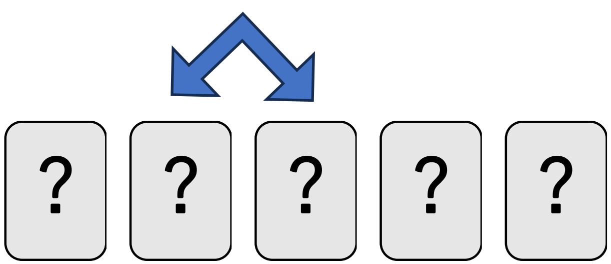
We want to remove $\boxed{2}\,\boxed{3}$

Niemi-Renvall AND protocol

1. Turn over 5^{th} card

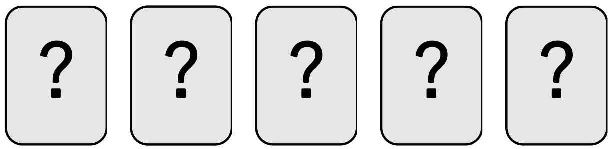


2. Swap 2^{nd} and 3^{rd} cards

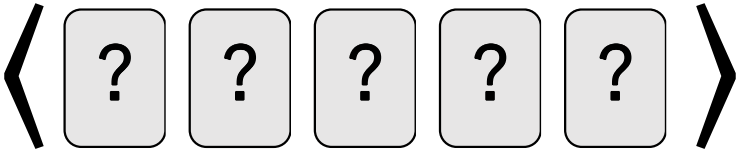


3. Remove $\boxed{2}\boxed{3}$ by random cuts

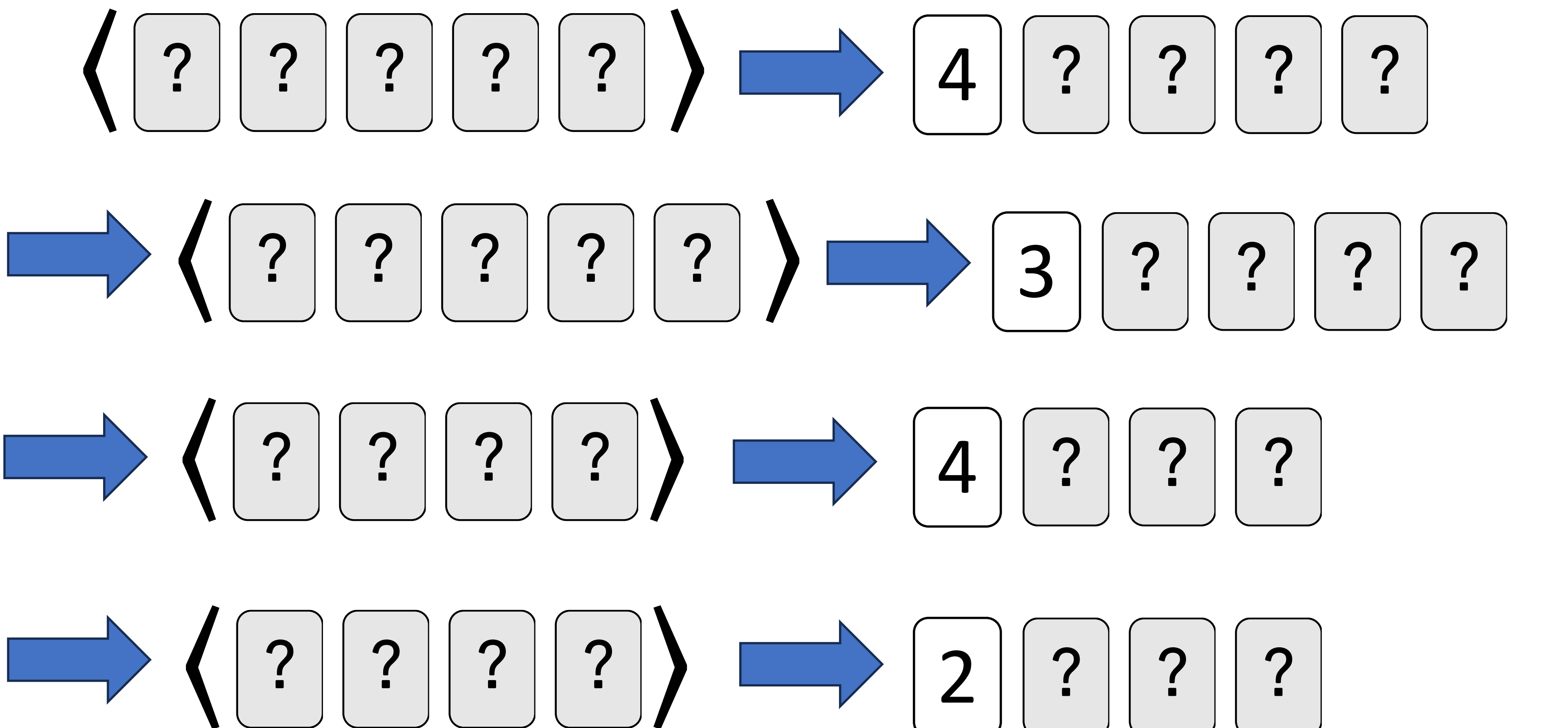
A ***random cut*** is a shuffle that cyclically shifts the sequence by a uniformly random offset.



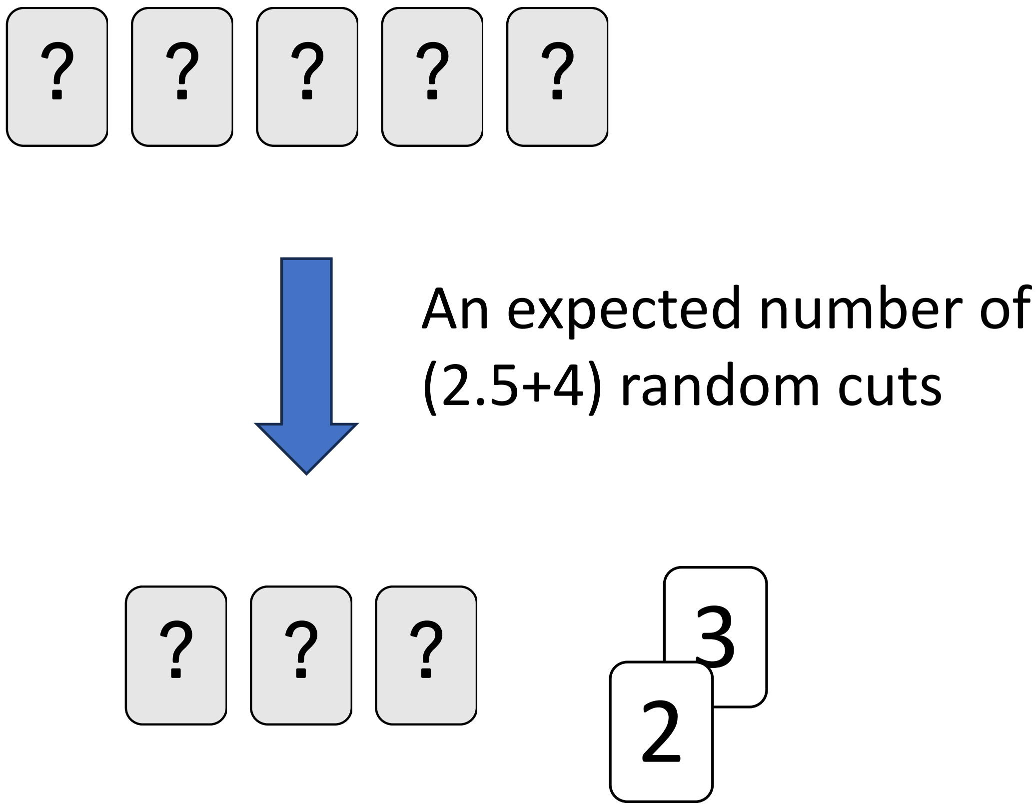
We use the following expression:



Reveal 1st card after applying a random cut until $\boxed{2}\,\boxed{3}$ are found



3. Remove 2 3 by random cuts



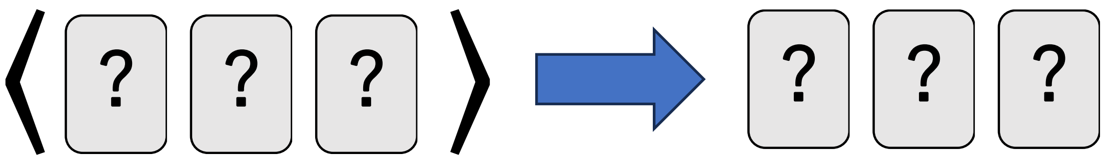
We can remove them without leaking any information.

1. Turn over 5^{th} card

2. Swap 2^{nd} and 3^{rd} cards

3. Remove $\boxed{2}\boxed{3}$ by random cuts

4. Apply a random cut



(a,b)	Sequence										
$(0,0)$	1	4	5	or	5	1	4	or	4	5	1
$(0,1)$	1	4	5	or	5	1	4	or	4	5	1 .
$(1,0)$	1	4	5	or	5	1	4	or	4	5	1
$(1,1)$	1	5	4	or	5	4	1	or	4	1	5

1. Turn over 5th card

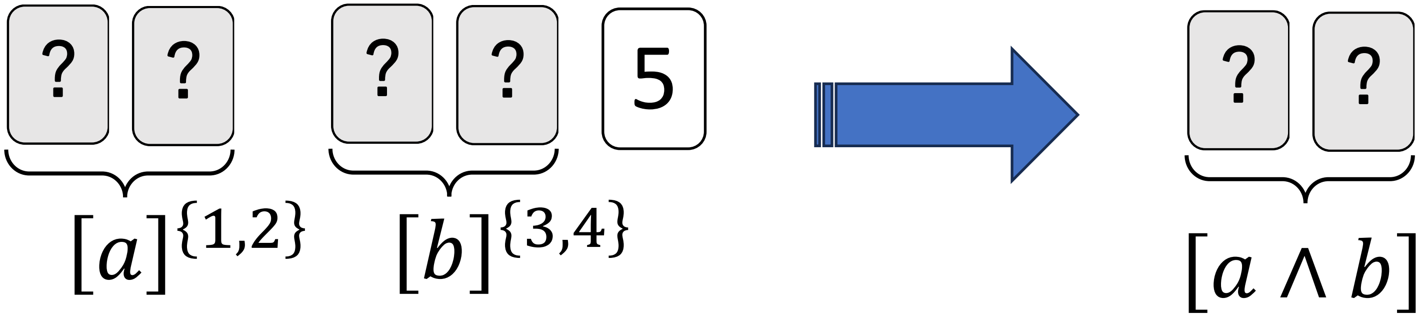
2. Swap 2nd and 3rd cards

3. Remove $\boxed{2}\boxed{3}$ by random cuts

4. Apply a random cut

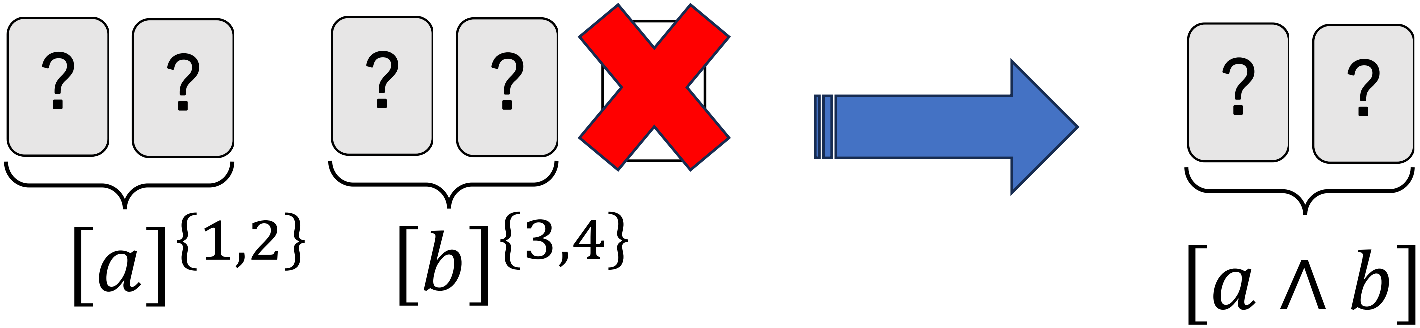
5. Reveal 1st card

$$\boxed{1}\ \underbrace{\boxed{?}\boxed{?}}_{[a\wedge b]^{\{4,5\}}} \quad \text{or} \quad \boxed{5}\ \underbrace{\boxed{?}\boxed{?}}_{[a\wedge b]^{\{1,4\}}} \quad \text{or} \quad \boxed{4}\ \underbrace{\boxed{?}\boxed{?}}_{[\overline{a\wedge b}]^{\{1,5\}}}$$



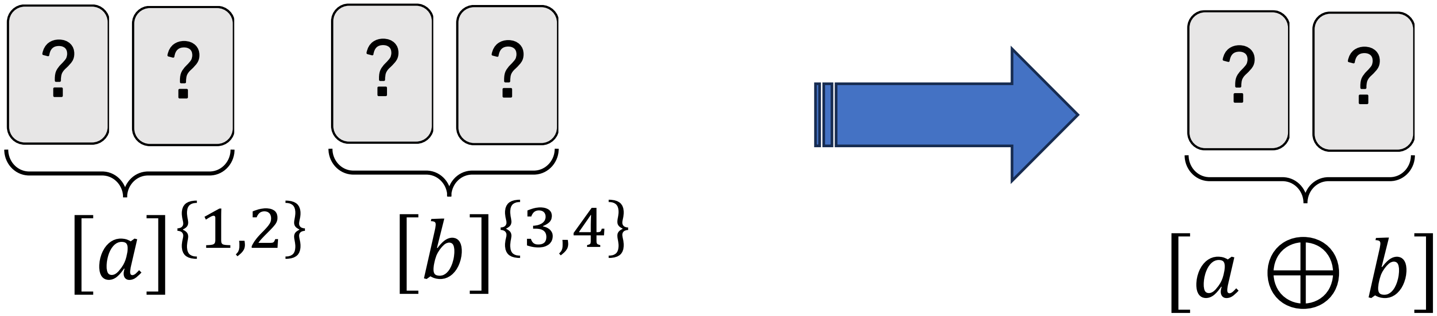
Function	# of cards	# of shuffles	Authors
AND	5	7.5 (exp)	Niemi-Renvall [NR99]

[NR99] Valtteri Niemi and Ari Renvall. Solitaire zero-knowledge. Fundam.Inf., 38(1,2):181–188, 1999



Function	# of cards	# of shuffles	Authors
AND	5	7.5 (exp)	Niemi-Renvall [NR99]
	4	6 (exp)	Koch et al. [KSK21]

[KSK21] Alexander Koch, Michael Schrempp, and Michael Kirsten. Card-based cryptography meets formal verification. New Gener. Com-put., 39(1):115–158, 2021.

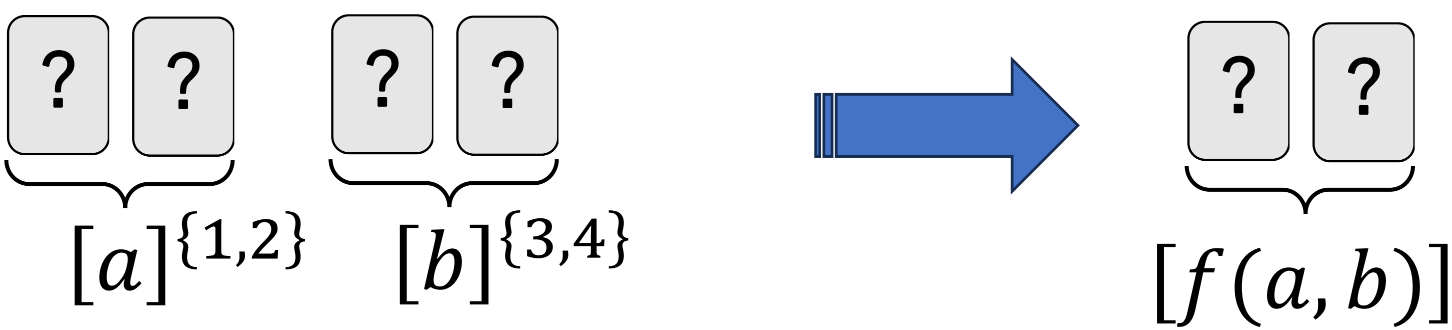


Function	# of cards	# of shuffles	Authors
AND	5	7.5 (exp)	Niemi-Renvall [NR99]
	4	6 (exp)	Koch et al. [KSK21]
XOR	4	7 (exp)	Niemi-Renvall [NR99]
	4	1	Mizuki [Miz16]

[Miz16] Takaaki Mizuki. Efficient and secure multiparty computations using a standard deck of playing cards. CANS 2026, vol.10052, LNCS, pp.484–499, 2016. Springer.

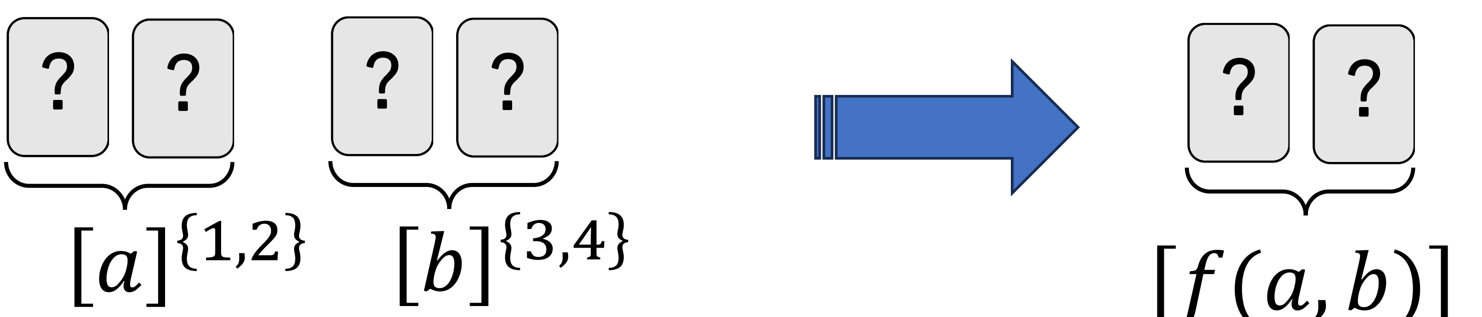
Function	# of cards	# of shuffles	Authors
AND	5	7.5 (exp)	Niemi-Renvall [NR99]
	4	6 (exp)	Koch et al. [KSK21]
XOR	4	7 (exp)	Niemi-Renvall [NR99]
	4	1	Mizuki [Miz16]

Card-minimal protocols for 2 inputs



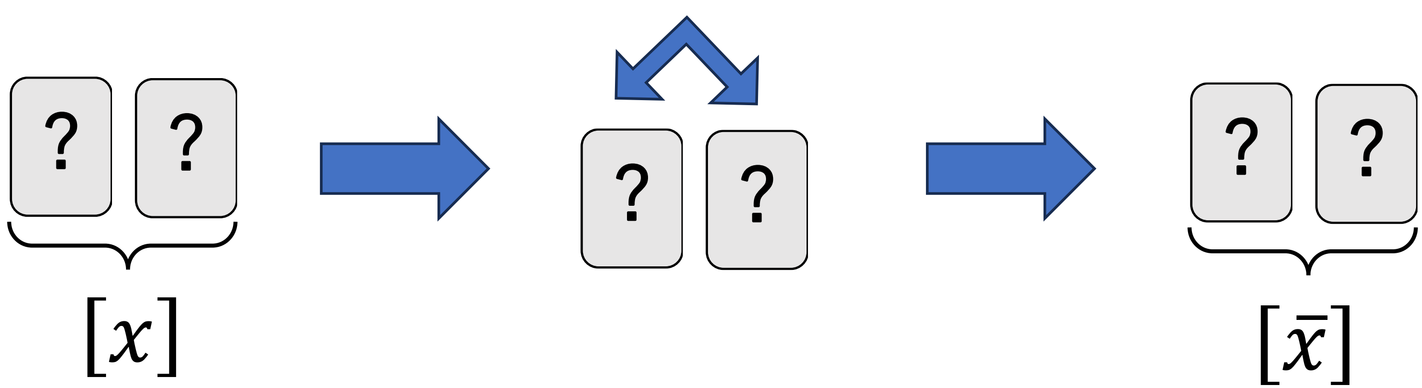
Function	# of cards	# of shuffles	Authors
AND	5	7.5 (exp)	Niemi-Renvall [NR99]
	4	6 (exp)	Koch et al. [KSK21]
XOR	4	7 (exp)	Niemi-Renvall [NR99]
	4	1	Mizuki [Miz16]

A 4-card protocol for any 2-input function $f: \{0,1\}^2 \to \{0,1\}$ can be constructed by AND, XOR, and NOT.



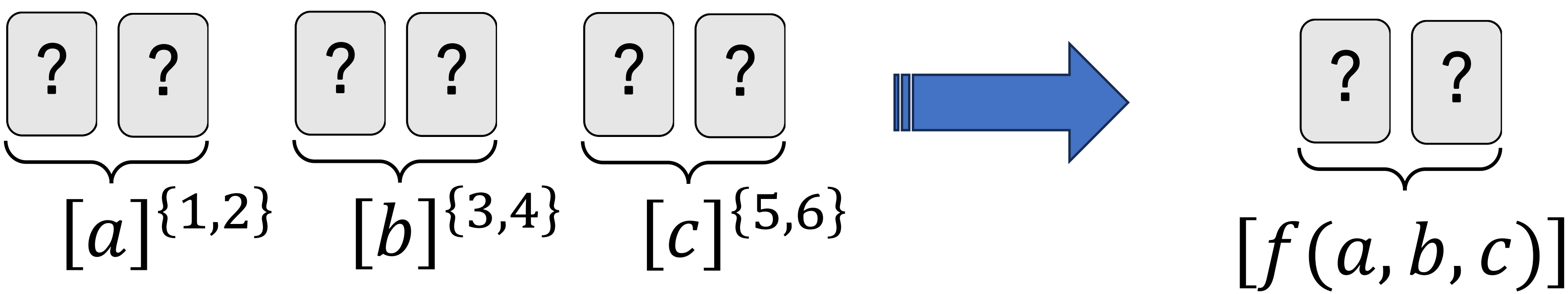
Function	# of cards	# of shuffles	Authors
AND	5	7.5 (exp)	Niemi-Renvall [NR99]
	4	6 (exp)	Koch et al. [KSK21]
XOR	4	7 (exp)	Niemi-Renvall [NR99]
	4	1	Mizuki [Miz16]

A 4-card protocol for any 2-input function $f: \{0,1\}^2 \rightarrow \{0,1\}$ can be constructed by AND, XOR, and **NOT**.

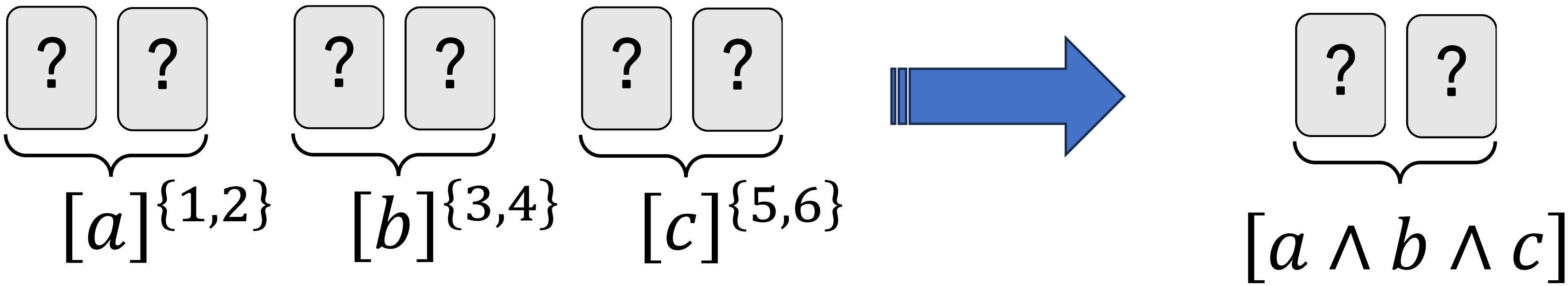


The problem of finding card-minimal protocols for **all 2-input** functions was already resolved.

How about **3-input** functions $f\colon \{0,1\}^3 \to \{0,1\}$?

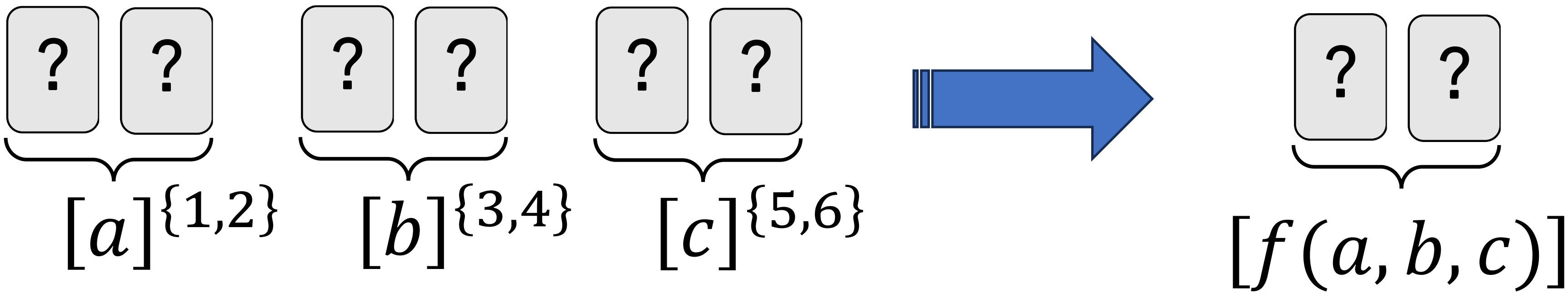


We want to construct a 6-card protocol (without any additional card except for 3 input commitments).



Function	# of cards	# of shuffles	Authors
3-AND	6	8.5 (exp)	Koyama et al. [KMMS21]

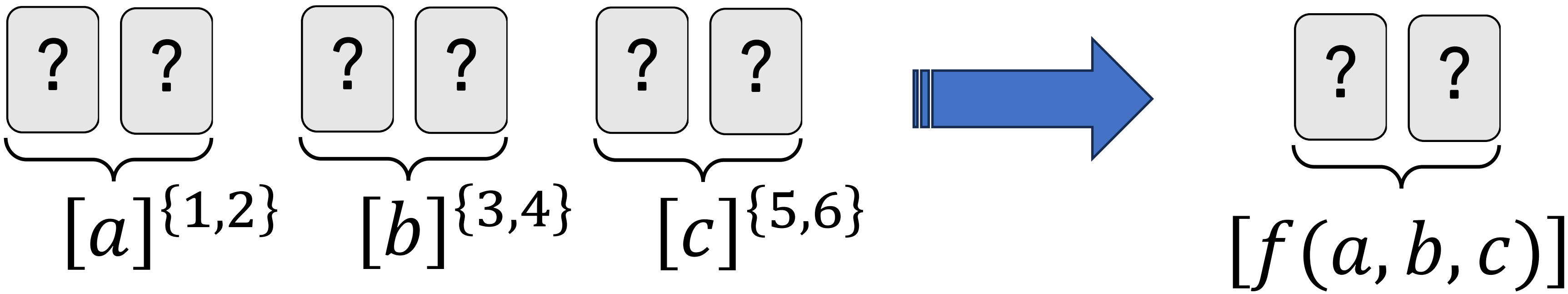
[KMMS21] Hiroto Koyama, Daiki Miyahara, Takaaki Mizuki, and Hideaki Sone. A secure three-input AND protocol with a standard deck of minimal cards. CSR 2021, vol.12730, LNCS, pp.242–256, 2021. Springer.



Function	# of cards	# of shuffles	Authors
3-AND	6	8.5 (exp)	Koyama et al. [KMMS21]
140 functions	6	8.5 (exp)	Haga et al. [HHMM22]

out of 256

[HHMM22] Rikuo Haga, Yuichi Hayashi, Daiki Miyahara, and Takaaki Mizuki. Card-minimal protocols for three-input functions with standard playing cards. AFRICACRYPT 2022, vol.13503, LNCS, pp.448–468, 2022. Springer



Function	# of cards	# of shuffles	Authors
3-AND	**6**	**8.5 (exp)**	**Koyama et al. [KMMS21]**
140 functions	**6**	**8.5 (exp)**	**Haga et al. [HHMM22]**

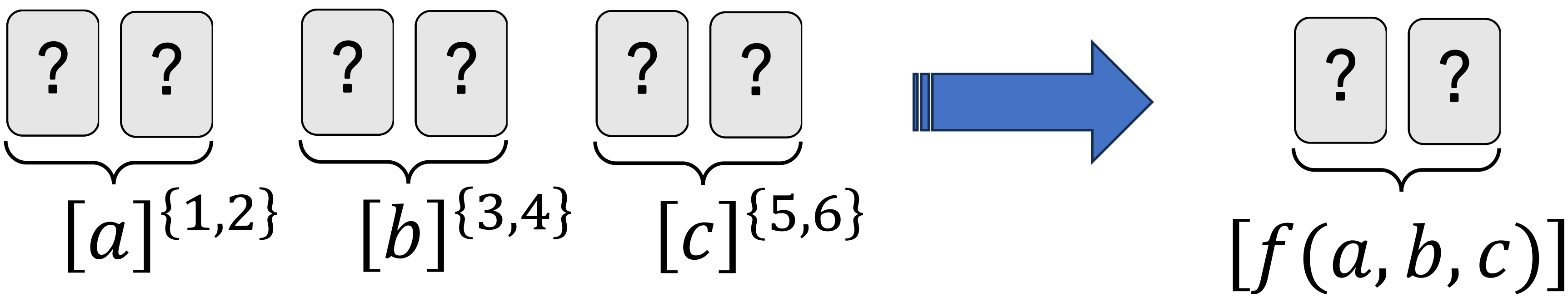
Open Problem:

Can 6-card protocols be constructed for the remaining **116** 3-input functions?

(Note that there are 256 3-input functions)

Our contribution

We show that, for the remaining **116** 3-input functions, 6-card protocols can be constructed.



Any function
$f:\{0,1\}^3 \rightarrow \{0,1\}$

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Idea

$\underbrace{\boxed{?}\ \boxed{?}}_{[a]^{\{1,2\}}}\ \underbrace{\boxed{?}\ \boxed{?}}_{[b]^{\{3,4\}}}$ $\xrightarrow{\text{Swap 2}^{\text{nd}}\text{ and 3}^{\text{rd}}\text{ cards}}$ $\boxed{?}\ \boxed{?}\ \boxed{?}\ \boxed{?}$

(a,b)				
(0,0)	1	2	3	4
(0,1)	1	2	4	3
(1,0)	2	1	3	4
(1,1)	2	1	4	3

$\boxed{1}\ \boxed{4} = 0$

$\boxed{1}\ \boxed{4} = 0$

$\boxed{1}\ \boxed{4} = 0$

$\boxed{1}\ \boxed{4} = 0$

Computes 0 (constant)

(a,b)				
(0,0)	1	3	2	4
(0,1)	1	4	2	3
(1,0)	2	3	1	4
(1,1)	2	4	1	3

$\boxed{1}\ \boxed{4} = 0$

$\boxed{1}\ \boxed{4} = 0$

$\boxed{1}\ \boxed{4} = 0$

$\boxed{4}\ \boxed{1} = 1$

Computes $a \wedge b$

Idea

$\underbrace{?\ ?}_{[a]^{\{1,2\}}}\ \underbrace{?\ ?}_{[b]^{\{3,4\}}}$

if $c = 1$

Swap 2nd and 3rd cards

Computes $a \wedge b$

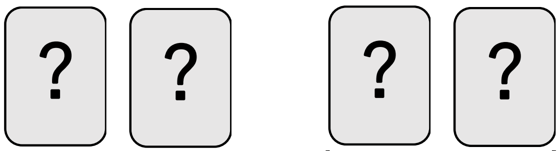
$\underbrace{?\ ?}_{[c]^{\{5,6\}}}$

if $c = 0$

Do nothing

Computes 0 (constant)

Idea

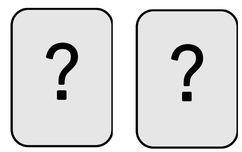


$[a]^{\{1,2\}}$ $[b]^{\{3,4\}}$

if $c = 1$

Swap 2nd and 3rd cards

Computes $a \wedge b$



$[c]^{\{5,6\}}$

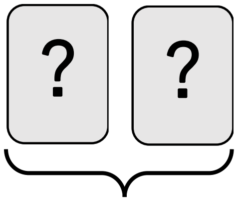
if $c = 0$

Do nothing

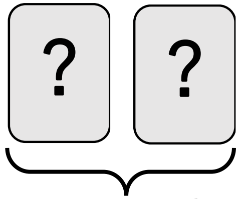
Computes 0 (constant)

$$\begin{cases} 0 & \text{if } c = 0 \\ a \wedge b & \text{if } c = 1 \end{cases}$$

Idea



$[a]^{\{1,2\}}$

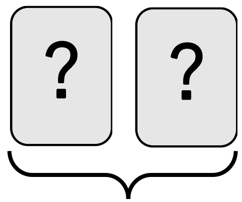


$[b]^{\{3,4\}}$

if $c = 1$

Swap 2nd and 3rd cards

Computes $a \wedge b$



$[c]^{\{5,6\}}$

if $c = 0$

Do nothing

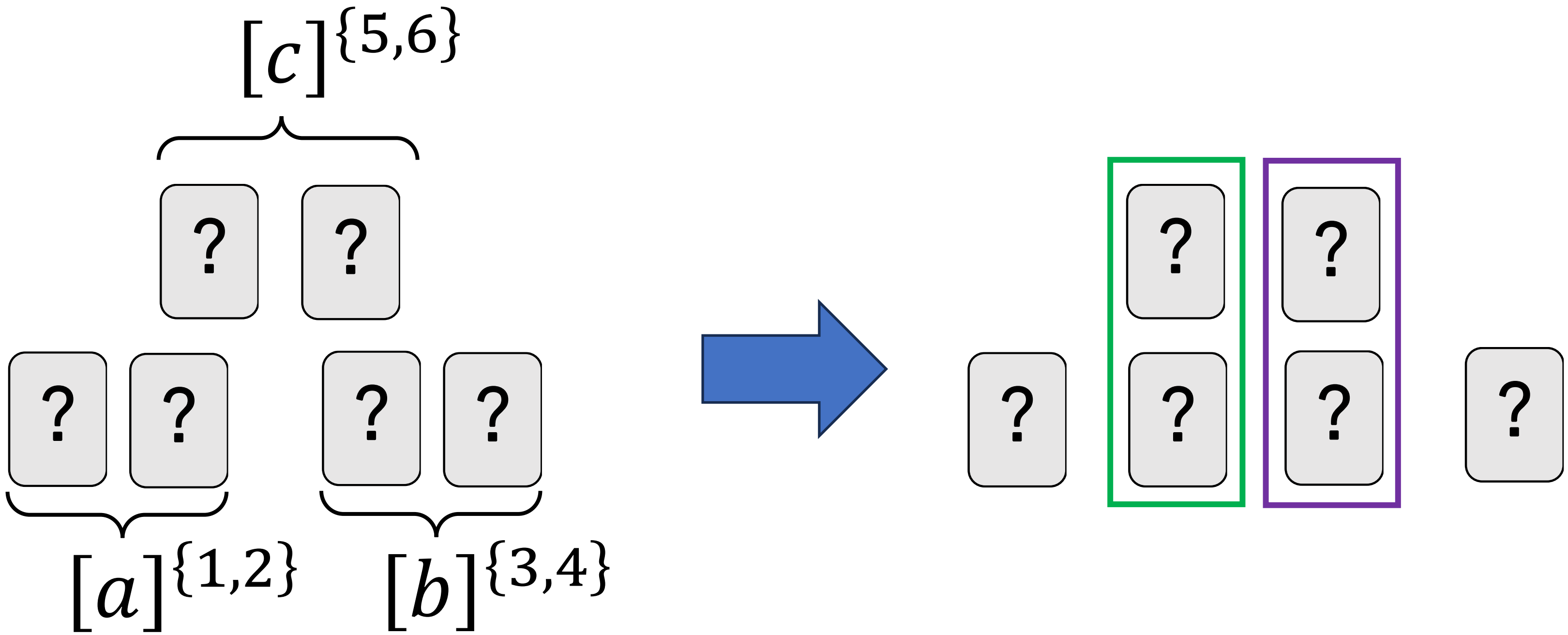
Computes 0 (constant)

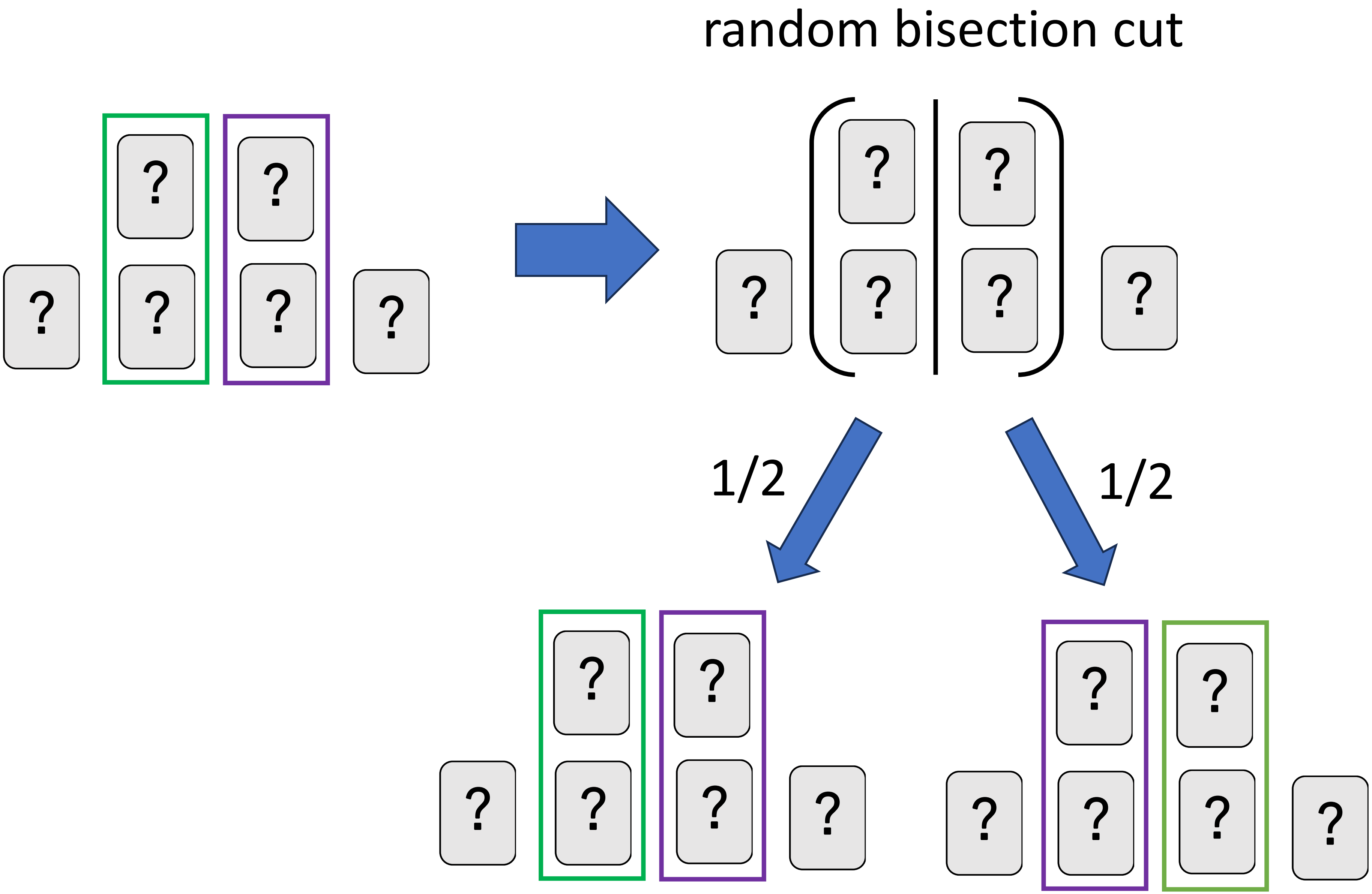
$$\begin{cases} 0 & \text{if } c = 0 \\ a \wedge b & \text{if } c = 1 \end{cases}$$

$$= \boldsymbol{a \wedge b \wedge c}$$

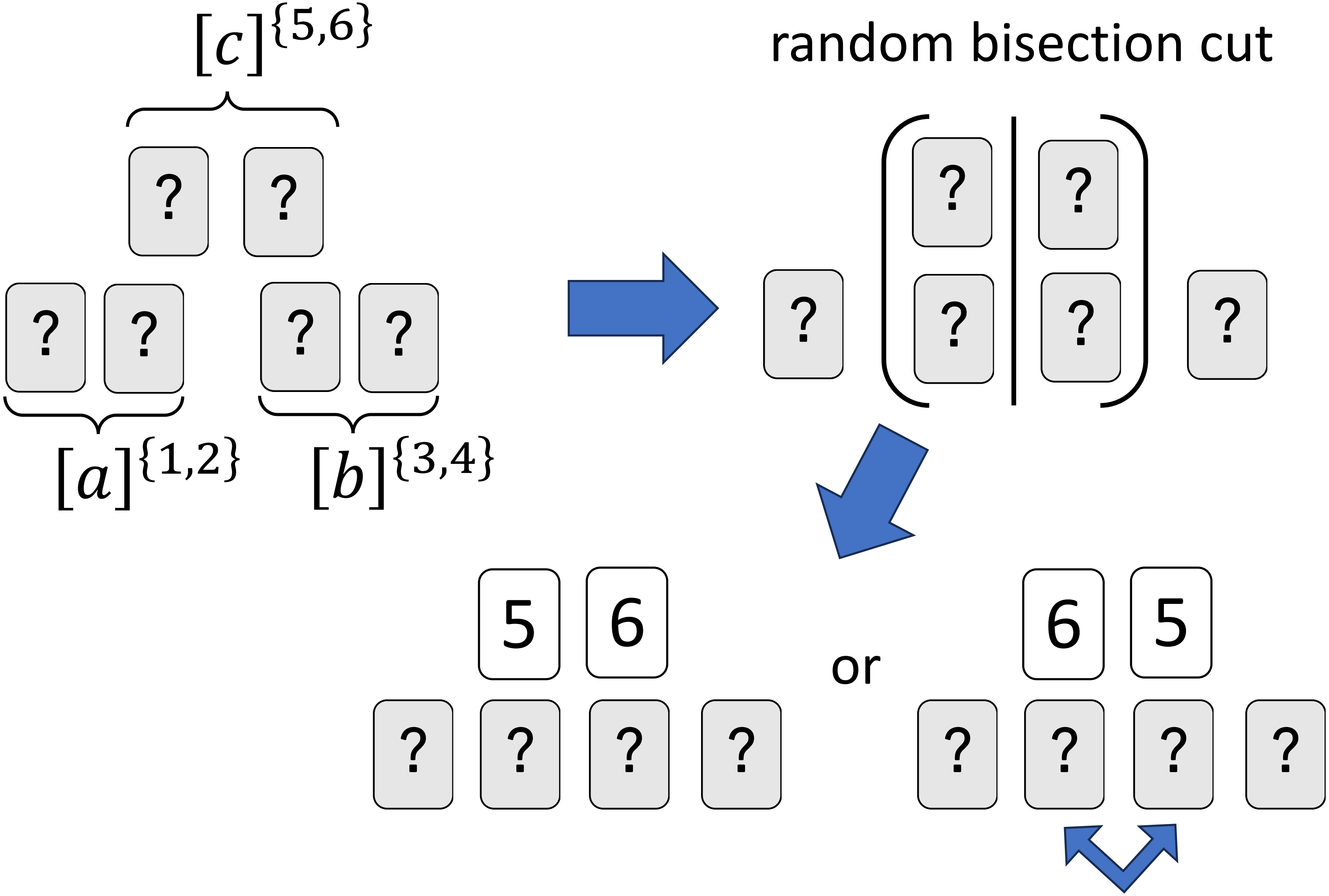
How to swap 2nd and 3rd only if $c = 1$

(without leaking any information)




random bisection cut
?
1/2
1/2

How to swap 2nd and 3rd only if $c = 1$



Outline of Haga et al.'s protocol

$$\underbrace{\boxed{?}\,\boxed{?}}_{[a]^{\{1,2\}}} \quad \underbrace{\boxed{?}\,\boxed{?}}_{[b]^{\{3,4\}}} \quad \underbrace{\boxed{?}\,\boxed{?}}_{[c]^{\{5,6\}}}$$

1. Swap some cards depending on c

$$\underbrace{\boxed{?}\,\boxed{?}}_{[c]}$$

$$\underline{\boxed{?}\,\boxed{?}\,\boxed{?}\,\boxed{?}}$$

2. Remove $\boxed{2}\,\boxed{3}$ by random cuts $\left\langle \boxed{?}\,\boxed{?}\,\boxed{?}\,\boxed{?}\,\boxed{5} \right\rangle$

3. Apply a random cut

4. Reveal 1st card

$$\underbrace{\boxed{?}\,\boxed{?}}_{[f(a,b,c)]}$$

Haga et al.'s method computes those in yellow.

$$f(a,b,c) = \begin{cases} g(a,b) & \text{if } c = 0 \\ h(a,b) & \text{if } c = 1 \end{cases}$$

140 out of 256 functions

$h \backslash g$	0	$a \wedge b$	$a \wedge \overline{b}$	a	$\overline{a} \wedge b$	b	$a \oplus b$	$a \vee b$	$\overline{a \vee b}$	$\overline{a \oplus b}$	$\overline{b}$	$a \vee \overline{b}$	$\overline{a}$	$\overline{a} \vee b$	$\overline{a \wedge b}$	1
0	✓	✓	✓	✓	✓	✓		✓	✓		✓	✓	✓	✓	✓	✓
$a \wedge b$	✓		✓	✓	✓	✓		✓	✓						✓	✓
$a \wedge \overline{b}$	✓	✓		✓	✓				✓		✓	✓		✓		✓
a	✓	✓	✓	✓		✓		✓			✓	✓	✓			✓
$\overline{a} \wedge b$	✓	✓	✓			✓			✓			✓	✓	✓		✓
b	✓	✓		✓	✓	✓		✓			✓		✓	✓		✓
$a \oplus b$																
$a \vee b$	✓	✓		✓		✓			✓			✓		✓	✓	✓
$\overline{a \vee b}$	✓	✓	✓		✓			✓			✓		✓		✓	✓
$\overline{a \oplus b}$																
$\overline{b}$	✓		✓	✓		✓			✓		✓	✓	✓		✓	✓
$a \vee \overline{b}$	✓		✓	✓	✓			✓			✓			✓	✓	✓
$\overline{a}$	✓			✓	✓	✓			✓		✓		✓	✓	✓	✓
$\overline{a} \vee b$	✓		✓		✓	✓		✓				✓	✓		✓	✓
$\overline{a \wedge b}$	✓	✓						✓	✓		✓	✓	✓	✓		✓
1	✓	✓	✓	✓	✓	✓		✓	✓		✓	✓	✓	✓	✓	✓

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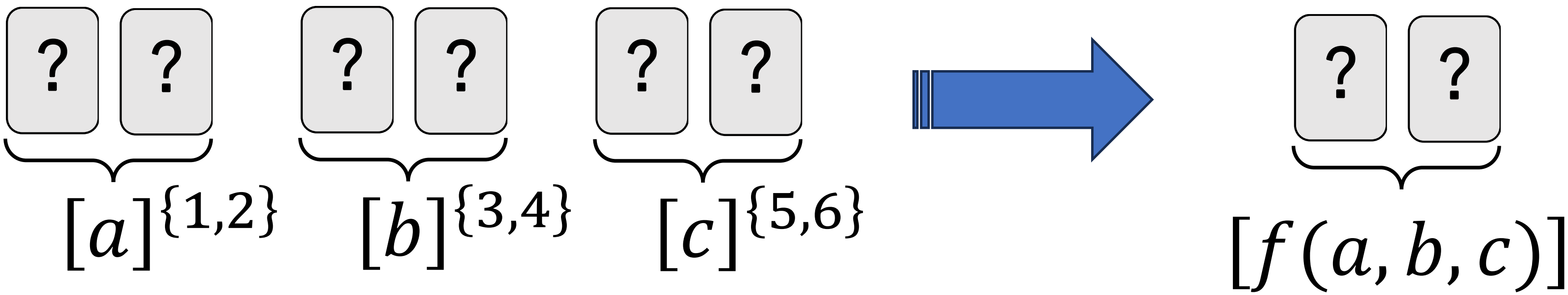
Assume that



Negation of inputs: $f(\overline{a},b,c), f(\overline{a},b,\overline{c})$, ...

Permutation of input order: $f(b,c,a), f(c,b,a)$, ...

Negation of output: $\overline{f}(a,b,c)$

All these functions can also be securely computed (because NOT and permutation of inputs are trivial).

Negation of inputs: $f(\overline{a}, b, c), f(\overline{a}, b, \overline{c})$, ...
Permutation of input order: $f(b, c, a), f(c, b, a)$, ...
Negation of output: $\overline{f}(a, b, c)$

We consider ***NPN equivalence classes***.

It suffices to deal with a representative function in each class.

It is well known that there are 14 NPN equivalence classes for 3-input functions.

It is well known that there are 14 NPN equivalence classes for 3-input functions.

Class	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14
Rep.	0	And3	$a \wedge b$	XorAnd	OrAnd	a	Onehot	Majority	Equality	Dot	Mux	AndXor	$a \oplus b$	Xor3

$$\begin{aligned}
\mathsf{And3}(a,b,c) &= a \wedge b \wedge c \\
\mathsf{XorAnd}(a,b,c) &= a \wedge (b \oplus c) \\
\mathsf{OrAnd}(a,b,c) &= a \wedge (b \vee c) \\
\mathsf{Onehot}(a,b,c) &= (a \wedge \overline{b} \wedge \overline{c}) \oplus (\overline{a} \wedge b \wedge \overline{c}) \oplus (\overline{a} \wedge \overline{b} \wedge c) \\
\mathsf{Majority}(a,b,c) &= (a \wedge b) \vee (b \wedge c) \vee (a \wedge c) \\
\mathsf{Equality}(a,b,c) &= (a \wedge b \wedge c) \oplus (\overline{a} \wedge \overline{b} \wedge \overline{c}) \\
\mathsf{Dot}(a,b,c) &= a \oplus (c \vee (a \wedge b)) \\
\mathsf{Mux}(a,b,c) &= (a \wedge b) \vee (\overline{a} \wedge c) \\
\mathsf{AndXor}(a,b,c) &= a \oplus (b \wedge c) \\
\mathsf{Xor3}(a,b,c) &= a \oplus b \oplus c\,.
\end{aligned}$$

$h \backslash g$	0	$a \wedge b$	$a \wedge \overline{b}$	a	$\overline{a} \wedge b$	b	$a \oplus b$	$a \vee b$	$\overline{a \vee b}$	$\overline{a \oplus b}$	$\overline{b}$	$a \vee \overline{b}$	$\overline{a}$	$\overline{a} \vee b$	$\overline{a \wedge b}$	1
0	#1	#2	#2	#3	#2	#3	#4	#5	#2	#4	#3	#5	#3	#5	#5	#6
$a \wedge b$	#2	#3	#4	#5	#4	#5	#10	#8	#9	#7	#10	#11	#10	#11	#12	#5
$a \wedge \overline{b}$	#2	#4	#3	#5	#9	#10	#7	#11	#4	#10	#5	#8	#10	#12	#11	#5
a	#3	#5	#5	#6	#10	#11	#12	#5	#10	#12	#11	#5	#13	#10	#10	#3
$\overline{a} \wedge b$	#2	#4	#9	#10	#3	#5	#7	#11	#4	#10	#10	#12	#5	#8	#11	#5
b	#3	#5	#10	#11	#5	#6	#12	#5	#10	#12	#13	#10	#11	#5	#10	#3
$a \oplus b$	#4	#10	#7	#12	#7	#12	#13	#7	#10	#14	#12	#10	#12	#10	#7	#4
$a \vee b$	#5	#8	#11	#5	#11	#5	#7	#3	#12	#10	#10	#4	#10	#4	#9	#2
$\overline{a \vee b}$	#2	#9	#4	#10	#4	#10	#10	#12	#3	#7	#5	#11	#5	#11	#8	#5
$\overline{a \oplus b}$	#4	#7	#10	#12	#10	#12	#14	#10	#7	#13	#12	#7	#12	#7	#10	#4
$\overline{b}$	#3	#10	#5	#11	#10	#13	#12	#10	#5	#12	#6	#5	#11	#10	#5	#3
$a \vee \overline{b}$	#5	#11	#8	#5	#12	#10	#10	#4	#11	#7	#5	#3	#10	#9	#4	#2
$\overline{a}$	#3	#10	#10	#13	#5	#11	#12	#10	#5	#12	#11	#10	#6	#5	#5	#3
$\overline{a} \vee b$	#5	#11	#12	#10	#8	#5	#10	#4	#11	#7	#10	#9	#5	#3	#4	#2
$\overline{a \wedge b}$	#5	#12	#11	#10	#11	#10	#7	#9	#8	#10	#5	#4	#5	#4	#3	#2
1	#6	#5	#5	#3	#5	#3	#4	#2	#5	#4	#3	#2	#3	#2	#2	#1

#1～#6, #8, #9, #11～#13 are covered.

#7, #10, and #14 are uncovered.

#7, #10, and #14 are uncovered.

#7:

Onehot $= (\boldsymbol{a} \wedge \overline{\boldsymbol{b}} \wedge \overline{\boldsymbol{c}}) \oplus (\overline{\boldsymbol{a}} \wedge \boldsymbol{b} \wedge \overline{\boldsymbol{c}}) \oplus (\overline{\boldsymbol{a}} \wedge \overline{\boldsymbol{b}} \wedge \boldsymbol{c})$

#10:

Dot $= \boldsymbol{a} \oplus (\boldsymbol{c} \vee (\boldsymbol{a} \wedge \boldsymbol{b}))$

#14:

Xor3 $= \boldsymbol{a} \oplus \boldsymbol{b} \oplus \boldsymbol{c}$

2-input XOR twice

The truly unresolved classes are #7 (Onehot) and #10 (Dot).
We will construct 6-card protocols for these two functions.

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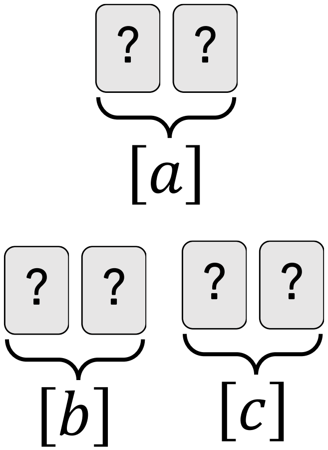
$$\mathrm{Onehot}(a,b,c) = \begin{cases} b \oplus c & \text{if } a = 0 \\ \overline{b \vee c} & \text{if } a = 1 \end{cases}$$

$$\underbrace{\boxed{?}\ \boxed{?}}_{[a]^{\{1,2\}}} \quad \underbrace{\boxed{?}\ \boxed{?}}_{[b]^{\{3,4\}}} \quad \underbrace{\boxed{?}\ \boxed{?}}_{[c]^{\{5,6\}}}$$

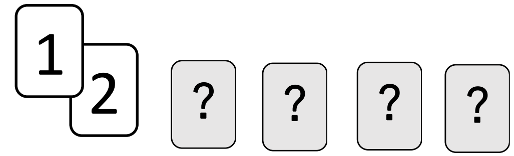
(b,c)				
(0,0)	3	4	5	6
(0,1)	3	4	6	5
(1,0)	4	3	5	6
(1,1)	4	3	6	5



Swap 1st and 4th cards only if $a = 0$:

(b,c)				
(0,0)	3	4	5	6
(0,1)	3	4	6	5
(1,0)	4	3	5	6
(1,1)	4	3	6	5

$a = 0$



(b,c)				
(0,0)	6	4	5	3
(0,1)	5	4	6	3
(1,0)	6	3	5	4
(1,1)	5	3	6	4

$a = 1$

(b,c)				
(0,0)	3	4	5	6
(0,1)	3	4	6	5
(1,0)	4	3	5	6
(1,1)	4	3	6	5

$a = 0$

$(\boldsymbol{b}, \boldsymbol{c})$				
(0,0)	6	4	5	3
(0,1)	5	4	6	3
(1,0)	6	3	5	4
(1,1)	5	3	6	4

$a = 1$

$(\boldsymbol{b}, \boldsymbol{c})$				
(0,0)	3	4	5	6
(0,1)	3	4	6	5
(1,0)	4	3	5	6
(1,1)	4	3	6	5

Find $\boxed{6}$ (or $\boxed{3}$):

$\left\langle \boxed{?}\ \boxed{?}\ \boxed{?}\ \boxed{?} \right\rangle$

$\boxed{1}\ \boxed{2}$

$\boxed{6}\ \boxed{?}\ \boxed{?}\ \boxed{?}$

$(\boldsymbol{b}, \boldsymbol{c})$				
(0,0)	6	4	5	3
(0,1)	6	3	5	4
(1,0)	6	3	5	4
(1,1)	6	4	5	3

$(\boldsymbol{b}, \boldsymbol{c})$				
(0,0)	6	3	4	5
(0,1)	6	5	3	4
(1,0)	6	4	3	5
(1,1)	6	5	4	3

1
2

6 ? ? ?

$a = 0$

(b, c)				
(0,0)	6	4	5	3
(0,1)	6	3	5	4
(1,0)	6	3	5	4
(1,1)	6	4	5	3

$a = 1$

(b, c)				
(0,0)	6	3	4	5
(0,1)	6	5	3	4
(1,0)	6	4	3	5
(1,1)	6	5	4	3

Move $\boxed{1}$:

2

6 ? 1 ? ?

(b, c)					
(0,0)	6	4	1	5	3
(0,1)	6	3	1	5	4
(1,0)	6	3	1	5	4
(1,1)	6	4	1	5	3

(b, c)					
(0,0)	6	3	1	4	5
(0,1)	6	5	1	3	4
(1,0)	6	4	1	3	5
(1,1)	6	5	1	4	3

Sentinel

$\boxed{2}$

$\boxed{6}$ $\boxed{?}$ $\boxed{1}$ $\boxed{?}$ $\boxed{?}$

$a = 0$

(b, c)					
(0,0)	6	4	1	5	3
(0,1)	6	3	1	5	4
(1,0)	6	3	1	5	4
(1,1)	6	4	1	5	3

$\boxed{1}\boxed{3} = 0$

$\boxed{3}\boxed{1} = 1$

$\boxed{3}\boxed{1} = 1$

$\boxed{1}\boxed{3} = 0$

$b \oplus c$

$a = 1$

(b, c)					
(0,0)	6	3	1	4	5
(0,1)	6	5	1	3	4
(1,0)	6	4	1	3	5
(1,1)	6	5	1	4	3

$\boxed{3}\boxed{1} = 1$

$\boxed{1}\boxed{3} = 0$

$\boxed{1}\boxed{3} = 0$

$\boxed{1}\boxed{3} = 0$

$\overline{b \vee c}$

Removing $\boxed{4}$ $\boxed{5}$ results in $\mathrm{Onehot}(a, b, c) = \begin{cases} b \oplus c & \text{if } a = 0 \\ \overline{b \vee c} & \text{if } a = 1 \end{cases}$

Our protocol for Onehot

$\underbrace{\boxed{?}\boxed{?}}_{[a]}$

$\underbrace{\boxed{?}\boxed{?}}_{[b]}$ $\underbrace{\boxed{?}\boxed{?}}_{[c]}$

1. Swap 1st and 4th cards if $a = 0$
2. Find $\boxed{6}$ (or $\boxed{3}$) $\boxed{6}\ \boxed{?}\ \boxed{?}\ \boxed{?}$
3. Insert $\boxed{1}$ $\boxed{6}\ \boxed{?}\ \boxed{1}\ \boxed{?}\ \boxed{?}$
4. Remove $\boxed{4}\boxed{5}$ $\boxed{?}\ \boxed{?}\ \boxed{?}$
5. Apply a random cut
6. Reveal 1st card

$\underbrace{\boxed{?}\boxed{?}}_{[\mathrm{Onehot}(a,b,c)]}$

Similarly, we can construct a protocol for **Dot** $= \boldsymbol{a} \oplus \left(\boldsymbol{c} \vee (\boldsymbol{a} \wedge \boldsymbol{b})\right)$

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We proposed 6-card protocols for Onehot and Dot, resolving the card-minimality problem for 3-input functions.

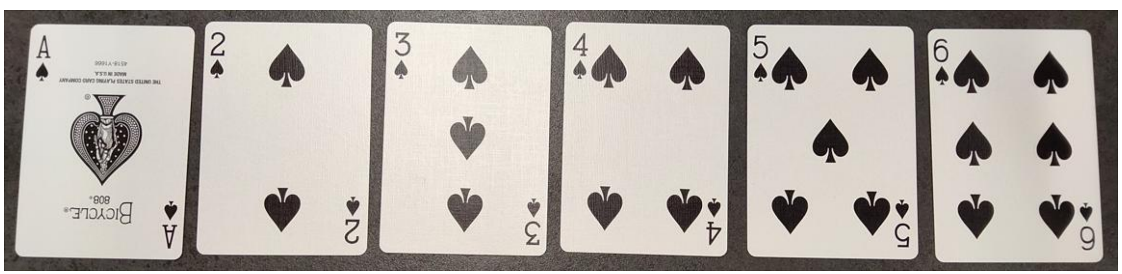
class	#1	#2	#3	#4	#5	#6	#7
Rep.	0	And3	$a \wedge b$	XorAnd	OrAnd	a	Onehot
Haga et al.	8.5	8.5	8.5	8.5	8.5	8.5	-
Trivial/ Known	0	-	6	7	-	0	-
This paper	-	-	-	-	-	-	10.5

class	#8	#9	#10	#11	#12	#13	#14
Rep.	Majority	Equality	Dot	Mux	AndXor	$a \oplus b$	Xor3
Haga et al.	8.5	8.5	-	8.5	8.5	8.5	-
Trivial/ Known	-	-	-	-	7	1	2
This paper	-	-	10.5	-	-	-	-

Future work:

- Reduce the number of shuffles
- Card-minimal problem for more than 3 inputs
  - As an upper bound, $2n + 8$ cards have been shown to be sufficient for any n-input function [SM19]

Enjoy secure computations with six cards!



[SM19] Kazumasa Shinagawa and Takaaki Mizuki. Secure computation ofany Boolean function based on any deck of cards. FAW 2019, vol.11458, LNCS, pp.63–75, 2019. Springer.